# Oscillations of 2DEG thermoelectric coefficients in magnetic field under microwave irradiation

A. E. Patrakov and I. I. Lyapilin\*
Institute of Metal Physics, UD of RAS, Yekaterinburg, Russia

It is known that under microwave irradiation, in 2D electron systems with high filling factors oscillations of longitudinal magnetoresistance appear in the range of magnetic fields where ordinary SdH oscillations are suppressed. In the present paper we propose a simple quasiclassical model of these new oscillations based on the Boltzmann kinetic equation. Our model also predicts similar oscillations in diffusion component of thermoelectric coefficients, which should be observable at low temperatures.

#### PACS numbers: 73.40.-c, 78.67.-n, 73.43.-f

### INTRODUCTION

The interest to theoretical investigations of non-linear transport phenomena in two-dimensional electron systems (2DES) has grown substantially due to new experimental results obtained in very clean 2DES samples. It has been discovered independently by two experimental groups [1, 2] that the resistance of two-dimensional highmobility electron gas in GaAs/AlGaAs heterostructures reveals a series of new features in its dependence on magnetic field H, temperature T, radiation power, etc. Under microwave irradiation, in 2DES with high filling factors oscillations of longitudinal magnetoresistance appear in the range of magnetic fields where ordinary SdH oscillations are suppressed. Under high-intensity irradiation, the minima of these oscillations become zero-resistance states. Unlike the SdH oscillations, which depend upon the ratio of the chemical potential  $\zeta$  to the cyclotron frequency  $\omega_c$ , these oscillations caused by irradiation depend upon the ratio of the radiation frequency  $\omega$  to the cyclotron one. A series of theoretical papers [3, 4, 5, 6] has been submitted that aim to explain these oscillations.

Let's mention important conditions of the experiments discussed above: the effect is seen at

$$\hbar/\tau \ll T \simeq \hbar\omega_{\rm c} \leq \hbar\omega \ll \zeta$$

where  $\tau$  is the transport relaxation time. It follows that the effect has a quasiclassical nature. Since the oscillations express themselves in the classical range of magnetic fields, it is reasonable to consider this effect using a model based on the Boltzmann kinetic equation. The Boltzmann kinetic equation is also capable of describing thermoelectric phenomena, and our model predicts oscillations of thermopower and Nernst — Ettingshausen coefficient controlled by the  $\omega/\omega_{\rm c}$  ratio. They should be observable at low temperatures.

However, in this paper only contribution of diffusion processes into thermoelectric coefficients is considered. At liquid helium temperatures, scattering of electrons upon nonequilibrium phonons and their mutual drag is also important. The contribution of processes involving

phonons to thermoelectric coefficients in the presence of microwave radiation will be considered elsewhere.

#### MODEL

Taking into account that the electrons are driven from the equilibrium both by the temperature gradient, the external dc electric field and the electric field of radiation, we have the following kinetic equation for electrons in the 2DES:

$$\frac{\boldsymbol{p}}{m} \frac{\partial f(\boldsymbol{r}, \boldsymbol{p})}{\partial \boldsymbol{r}} + \frac{\partial f(\boldsymbol{r}, \boldsymbol{p})}{\partial t} \bigg|_{E_{ac}} - e \boldsymbol{E}_{dc} \frac{\partial f(\boldsymbol{r}, \boldsymbol{p})}{\partial \boldsymbol{p}} - \frac{e}{mc} \left[ \boldsymbol{p} \times \boldsymbol{H} \right] \frac{\partial f(\boldsymbol{r}, \boldsymbol{p})}{\partial \boldsymbol{p}} = I_{st} [f(\boldsymbol{r}, \boldsymbol{p})]. \quad (1)$$

Here  $E_{\rm dc}$  is the strength of the external dc field. The right hand side is the collision integral. We assume that the main electron scattering mechanism is the elastic scattering on impurities:

$$I_{st}[f(\mathbf{r}, \mathbf{p})] = -\frac{f(\mathbf{r}, \mathbf{p}) - f_0(\mathbf{r}, \mathbf{p})}{\tau},$$
 (2)

where  $f_0(\boldsymbol{r}, \boldsymbol{p})$  is the equilibrium distribution function,  $\tau$  is the relaxation time.  $\frac{\partial f(\boldsymbol{r}, \boldsymbol{p})}{\partial t}\Big|_{E_{\rm ac}}$  is the rate of distribution function change due to transitions between Landau levels caused by microwave radiation:

$$\left. \frac{\partial f(\boldsymbol{r}, \boldsymbol{p})}{\partial t} \right|_{E_{ac}} = \sum_{\boldsymbol{p}'} w_{\boldsymbol{p}\boldsymbol{p}'}(f(\boldsymbol{r}, \boldsymbol{p}') - f(\boldsymbol{r}, \boldsymbol{p})), \quad (3)$$

where  $w_{pp'}$  is the probability of the electron transition from the state with the kinetic momentum p to the state with the kinetic momentum p' in a unit of time due to microwave radiation.

Up to the first order on thermodynamic forces, the rate of distribution function change resulting from diffusion due to the temperature gradient is:

$$\frac{\boldsymbol{p}}{m}\frac{\partial f(\boldsymbol{r},\boldsymbol{p})}{\partial \boldsymbol{r}} = -\frac{\boldsymbol{p}}{m}T\frac{\partial f_0(T(\boldsymbol{r}),\varepsilon(\boldsymbol{p}))}{\partial \varepsilon(\boldsymbol{p})}\frac{\partial}{\partial \boldsymbol{r}}\left(\frac{\varepsilon(\boldsymbol{p})-\zeta}{T(\boldsymbol{r})}\right). \tag{4}$$

The non-equilibrium distribution function can be represented in the following form:

$$f(\mathbf{r}, \mathbf{p}) = f_0(T(\mathbf{r}), \varepsilon(\mathbf{p})) + \mathbf{p}\mathbf{g}(\varepsilon(\mathbf{p})), \tag{5}$$

where  $f_0(T(\mathbf{r}), \varepsilon(\mathbf{p}))$  is the equilibrium distribution function and  $\mathbf{g}(\varepsilon(\mathbf{p}))$  is an unknown function that depends only on electron energy.

The main effect of the ac electric field amounts to changes in electron energy (and not its momentum).

# DISTRIBUTION FUNCTION AND CURRENT DENSITY

Taking all simplifications mentioned above into account and linearizing upon thermodynamic forces, we can express the kinetic equation in the following form:

$$\sum_{\mathbf{p}'} w_{\mathbf{p}\mathbf{p}'}(\mathbf{g}(\varepsilon') - \mathbf{g}(\varepsilon)) - \frac{1}{m} \mathcal{F}_{dc} \frac{\partial f_0(T, \varepsilon)}{\partial \varepsilon} - \frac{e}{mc} \left[ \mathbf{H} \times \mathbf{g}(\varepsilon) \right] + \frac{\mathbf{g}(\varepsilon)}{\tau} = 0, \quad (6)$$

where we denoted

$$\mathcal{F}_{dc} = e\mathbf{E}_{dc} + \frac{\varepsilon - \zeta}{T}\nabla T. \tag{7}$$

We shall search for a solution of (6) in the form of a power series on  $w_{pp'}$ . Up to the linear terms, we have:

$$\mathbf{g}(\varepsilon) = \mathbf{g}_{0}(\varepsilon) - \frac{\tau^{2}}{(1 + \omega_{c}^{2} \tau^{2})^{2}} \frac{1}{m} \times ((1 - \omega_{c}^{2} \tau^{2}) \mathcal{F}_{dc} + 2\tau [\boldsymbol{\omega}_{c} \times \mathcal{F}_{dc}]) \times \sum_{\pm} \rho(\varepsilon \pm \hbar \omega) w_{\pm} \left( \frac{\partial f_{0}(\varepsilon \pm \hbar \omega)}{\partial \varepsilon} - \frac{\partial f_{0}(\varepsilon)}{\partial \varepsilon} \right), \quad (8)$$

where

$$\mathbf{g}_0 = \frac{\tau}{1 + \omega_c^2 \tau^2} \frac{1}{m} (\mathcal{F}_{dc} + \tau [\boldsymbol{\omega}_c \times \mathcal{F}_{dc}]) \frac{\partial f_0(\varepsilon)}{\partial \varepsilon}.$$
 (9)

Here  $\omega_{\rm c}=(eH)/(mc)$  and it has been taken into account that  $\nabla T$ ,  $E_{\rm dc}\perp H$ . The probability of transition with absorption or emission of a photon is  $w_{\pm}\propto E_{\rm ac}^2\ell^2 \left|\langle n\pm 1|x|n\rangle\right|^2\propto E_{\rm ac}^2n/\omega_{\rm c}\propto E_{\rm ac}^2\varepsilon_n/\omega_{\rm c}^2$ , where  $E_{\rm ac}$  is the amplitude of the ac electric field with the frequency of  $\omega$ ,  $\rho(\varepsilon)$  is the density of states, and  $\ell=\sqrt{\hbar/(m\omega_{\rm c})}$  is the magnetic length.

Using the correction to the distribution function calculated above we find the current density caused by radiation:

$$\mathbf{j} = -\frac{2e}{(2\pi\hbar)^2 \rho_0} \int d\mathbf{p} \ \mathbf{p}(\mathbf{p}\mathbf{g}(\varepsilon(\mathbf{p}))), \tag{10}$$

where  $\rho_0 = m/(2\pi\hbar^2)$  is the density of states without the magnetic field for one spin direction.

The current can originate from the dc electric field or due to the temperature gradient. In linear theory, the relation of current density with thermodynamic forces is:

$$j_k = \sigma_{kl} E_{\text{dc},l} - \beta_{kl} \nabla_l T \tag{11}$$

Inserting (8) into (10) and putting  $\nabla T = 0$ , we obtain for diagonal components of the conductivity tensor:

$$\sigma_{\rm xx}^{\rm ph} = -\frac{2e^2}{m} \frac{\tau^2(\omega_{\rm c}^2 \tau^2 - 1)}{(1 + \omega_{\rm c}^2 \tau^2)^2} \sum_{\pm} \int_0^{\infty} d\varepsilon \, \varepsilon \times w_{\pm} \rho(\varepsilon) \rho(\varepsilon \pm \hbar \omega) \left( \frac{\partial f_0(\varepsilon \pm \hbar \omega)}{\partial \varepsilon} - \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) \quad (12)$$

Taking the energy integral under the assumption of strong degeneracy of electrons and using an explicit expression for the density of states [7]

$$\rho(\varepsilon) = \rho_0 (1 - \delta \cos(\frac{2\pi\varepsilon}{\hbar\omega_c})),$$

$$\delta = 2e^{-\pi/(\omega_c \tau_f)} \ll 1$$
(13)

( $\tau_f$  is a single-particle lifetime without magnetic field) we obtain the following expression for longitudinal photoconductivity:

$$\sigma_{\rm xx}^{\rm ph} - \sigma_{\rm xx}^{\rm ph, SdH} \propto \frac{\omega^2 \tau^2 (\omega_{\rm c}^2 \tau^2 - 1)}{\omega_{\rm c}^2 (1 + \omega_{\rm c}^2 \tau^2)^2} \delta^2 \cos \frac{2\pi\omega}{\omega_{\rm c}}$$
(14)

For simplicity, we denote all factors in  $\sigma_{\rm xx}^{\rm ph} - \sigma_{\rm xx}^{\rm ph}$ , SdH except  $(\omega_{\rm c}^2 \tau^2 - 1) \cos(2\pi\omega/\omega_{\rm c})$  with the letter A (A is proportional to the microwave power):

$$\sigma_{\rm xx}^{\rm ph} - \sigma_{\rm xx}^{\rm ph, SdH} = A(\omega_c^2 \tau^2 - 1) \cos \frac{2\pi\omega}{\omega_c}$$
 (15)

Performing similar calculations, one obtains for Hall photoconductivity:

$$\sigma_{\rm xy}^{\rm ph} - \sigma_{\rm xy}^{\rm ph, SdH} = 2A\omega_{\rm c}\tau\cos\frac{2\pi\omega}{\omega_{\rm c}}$$
 (16)

with the same factor A as above.

Then we calculate the change of  $\beta$  tensor components due to microwave irradiation. To do this, we insert (8) into (10) and put  $E_{\rm dc} = 0$ . The result, under the same assumptions, is:

$$\beta_{\rm xx}^{\rm ph} = \frac{2A\zeta(1 - \omega_{\rm c}^2 \tau^2)}{eT} \cos\left(\frac{2\pi\omega}{\omega_{\rm c}}\right) \tag{17}$$

$$\beta_{xy}^{ph} = -\frac{4A\zeta\omega_{c}\tau}{eT}\cos\left(\frac{2\pi\omega}{\omega_{c}}\right)$$
 (18)

In thermoelectric experiments, one usually does not allow the current to flow in the sample, applies the temperature gradient, and measures the voltage between the opposite edges of the sample. To find the electric field that develops due to the temperature gradient in such conditions, one should solve the following system of equations:

$$j_{x} = \sigma_{xx}^{0} E_{dc, x} + \sigma_{xx}^{ph} E_{dc, x} + \sigma_{xy}^{0} E_{dc, y} + \sigma_{xy}^{ph} E_{dc, y} - \beta_{xx}^{o} \nabla_{x} T - \beta_{xx}^{ph} \nabla_{x} T = 0$$
 (19)

$$j_{y} = \sigma_{xx}^{0} E_{dc, y} + \sigma_{xx}^{ph} E_{dc, y} - \sigma_{xy}^{0} E_{dc, x} - \sigma_{xy}^{ph} E_{dc, x} + \beta_{xy}^{0} \nabla_{x} T + \beta_{xy}^{ph} \nabla_{x} T = 0, \quad (20)$$

where  $\sigma^0$  and  $\beta^0$  denote the corresponding coefficients without microwave irradiation.

Solving this linear system and taking into account the following relations:

$$\sigma_{xy}^0 = -\omega_c \tau \sigma_{xx}^0,$$

$$\beta_{xy}^0 = -\omega_c \tau \beta_{xx}^0,$$

and also  $\zeta \gg T$ , we find:

$$E_{\rm dc, x} = \left(\frac{\beta_{\rm xx}^0}{\sigma_{\rm xx}^0} + \frac{2\zeta}{eT\sigma_{\rm xx}^0}A\cos\left(\frac{2\pi\omega}{\omega_{\rm c}}\right) + O(A^2)\right)\nabla_{\rm x}T$$
(21)

$$E_{\rm dc, y} = \left(\frac{2\zeta\omega_{\rm c}\tau}{eT\sigma_{\rm xx}^0}A\cos\left(\frac{2\pi\omega}{\omega_{\rm c}}\right) + O(A^2)\right)\nabla_{\rm x}T \quad (22)$$

## CONCLUSION

One can see from Eqs. (21), (22) that the diffusion components of thermoelectric coefficients (namely, thermopower and the Nernst — Ettingshausen coefficient) in the presence of microwave irradiation should oscillate with the same period in 1/B as magnetoresistance does. This effect should be better observed at low temperatures, when the thermopower is low by itself, and the correction is large due to the 1/T factor, unusual for diffusion processes.

- \* Electronic address: Lyapilin@imp.uran.ru
- M. A. Zudov, R. R. Du, L. N. Pfeiffer, K. W. West Phys. Rev. Lett.90 046807 (2003)
- [2] R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti Nature 420 646 (2002)
- [3] J. Shi and X. C. Xie Phys. Rev. Lett.91 086801 (2003)
- [4] A. C. Durst et. al. Phys. Rev. Lett. 91 086803 (2003)
- [5] V. Ryzhii and V. Vyurkov arXiv:cond-mat/0305199
- [6] S. A. Mikhailov 2003 arXiv:cond-mat/0303130v1
- [7] T. Ando, A. B. Fowler and F. Stern Rev. Mod. Phys. 54 437 (1982)